Chapter 26

1

Capacitance And Dielectrics

1

Lecture 02

2

- Capacitors Combination
- Energy Stored In A Capacitor

2

Capacitors combination

3

CAPACITORS IN PARALLEL

CAPACITORS IN SERIES

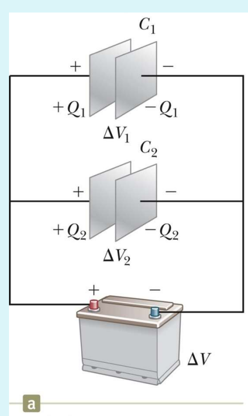
3

Capacitors in Parallel

4

When capacitors are first connected in the circuit, electrons are transferred from the left plates through the battery to the right plate, leaving the left plate positively charged and the right plate negatively charged.

The flow of charges ceases when the voltage across the capacitors equals that of the battery.



4

Capacitors in Parallel, 2

5

The potential difference across the capacitors is the same.

And each is equal to the voltage of the battery

$$\Delta V_1 = \Delta V_2 = \Delta V$$

ΔV is the battery terminal voltage

The total charge is equal to the sum of the charges on the capacitors.

$$Q_{tot} = Q_1 + Q_2$$

5

Capacitors in Parallel, 3

6

The capacitors can be replaced with one capacitor with a capacitance of C_{eq}

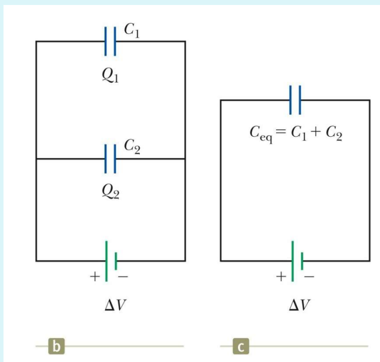
The *equivalent capacitor* must have exactly the same external effect on the circuit as the original capacitors.

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

The equivalent capacitance of a parallel combination of capacitors is greater than any of the individual capacitors.

When n capacitors of the same magnitude (C) are connected in parallel:

$$C_{eq} = nC$$



6

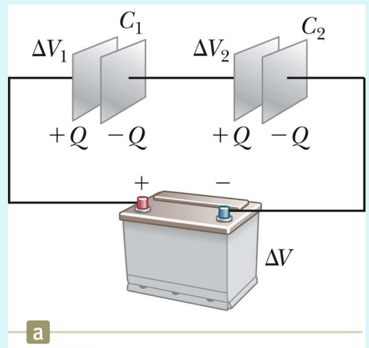
Capacitors in Series

7

When a battery is connected to the circuit, electrons are transferred from the left plate of C_1 to the right plate of C_2 through the battery.

As this negative charge accumulates on the right plate of C_2, an equivalent amount of negative charge is removed from the left plate of C_2, leaving it with an excess positive charge.

All of the right plates gain charges of $-Q$ and all the left plates have charges of $+Q$.



Mustafa Al-Zyout - Philadelphia University 10/5/2025

7

Capacitors in Series, cont.

8

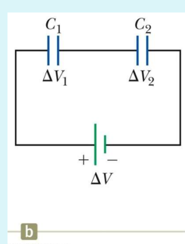
The charges are all the same.

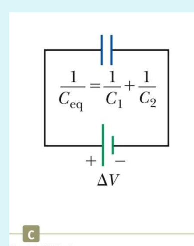
$$Q_1 = Q_2 = Q$$

The potential differences add up to the battery voltage.

$$\Delta V_{total} = \Delta V_1 + \Delta V_2$$

An equivalent capacitor can be found that performs the same function as the series combination.





Mustafa Al-Zyout - Philadelphia University 10/5/2025

8

Capacitors in Series, final

9

The equivalent capacitance is:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$$

The equivalent capacitance of a series combination is always less than any individual capacitor in the combination.

For n capacitors of the same magnitude (C) are in series:

$$C_{eq} = \frac{C}{n}$$

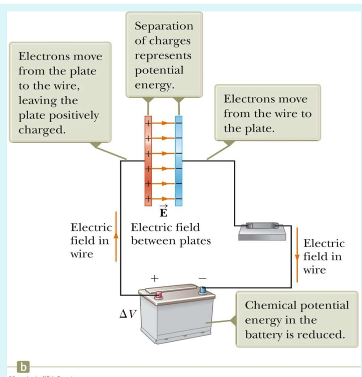
Energy in a Capacitor

10

Before the switch is closed, the energy is stored as chemical energy in the battery.

When the switch is closed, the energy is transformed from chemical potential energy to electric potential energy.

A capacitor can be described as a device that stores energy as well as charge.



Energy Stored in a Capacitor

11

Assume the capacitor is being charged and, at some point, has a charge dq on it.

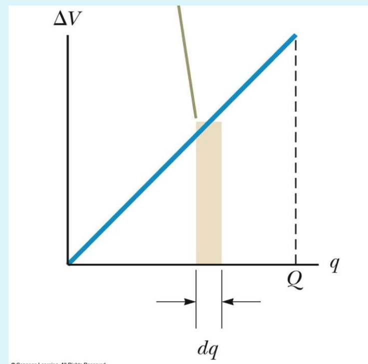
The work needed to transfer a charge from one plate to the other is

$$dW = (dq)(\Delta V) = \frac{q}{C}dq$$

The work required is the area of the tan rectangle.

The total work required is

$$W = \int_0^Q \frac{q}{C}dq = \frac{Q^2}{2C}$$



$$\text{slope} = \frac{\Delta V}{Q} = \frac{1}{C}$$

11

Energy

12

The work done in charging the capacitor appears as electric potential energy U:

$$U = \frac{Q^2}{2C}$$

$$U = \frac{1}{2}Q(\Delta V)$$

$$U = \frac{1}{2}C(\Delta V)^2$$

This applies to a capacitor of any geometry.

The energy stored increases as the charge increases and as the potential difference increases.

12

Energy

13

The energy can be considered to be **stored in the electric field** .

For a parallel-plate capacitor, the energy can be expressed in terms of the field as

$$U = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}\frac{A\epsilon_\circ}{d}(Ed)^2$$

$$\boxed{U = \frac{1}{2}\epsilon_\circ(Ad)E^2}$$

13

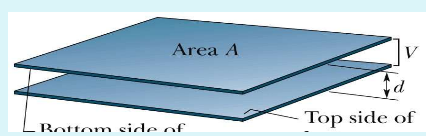
Energy density

14

It can also be expressed in terms of the **energy density ($\boldsymbol{u}$)** (energy per unit volume) (in units of J/m^3)

$$u = \frac{U}{\text{Volume}} = \frac{U}{Ad} = \frac{\frac{1}{2}\epsilon_\circ(Ad)E^2}{Ad}$$

$$\boxed{u = \frac{1}{2}\epsilon_\circ E^2}$$



The expression is generally valid regardless of the source of the electric field.
The energy density in *any* electric field is proportional to the square of the magnitude of the electric field at a given point.

14

Capacitors in parallel and in series

Friday, 29 January, 2021 21:01

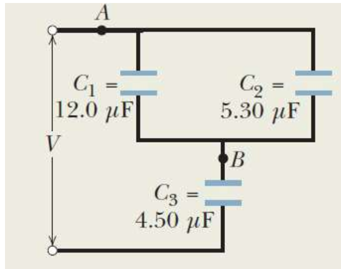
Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.

J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.

H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.

H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.



- Find the equivalent capacitance for the combination of capacitances shown in the figure, across which potential difference (V) is applied.

  Assume $C_1 = 12\ \mu F$, $C_2 = 5.3\ \mu F$ and $C_3 = 4.5\ \mu F$.

- The potential difference applied to the input terminals is $V = 12.5\ V$. What is the charge on C_1 ?and
- The energy stored in C_1 ?

- capacitor 1 and capacitor 2 are in parallel, and their equivalent capacitance C_{12} is:

$$C_{12} = C_1 + C_2 = 12.0\mu F + 5.30\mu F = 17.3\mu F$$

capacitor 12 and capacitor 3 are in series, and we can replace them with their equivalent C_{123} ("one two three"), as:

$$\frac{1}{C_{123}} = \frac{1}{C_{12}} + \frac{1}{C_3} = \frac{1}{17.3\mu F} + \frac{1}{4.50\mu F} = 0.280\mu F^{-1},$$

from which

$$C_{123} = \frac{1}{0.280\mu F^{-1}} = 3.57\mu F.$$

- Series capacitors have the same charge as their equivalent capacitor.

Parallel capacitors have the same potential difference as their equivalent capacitor.

Starting with the equivalent capacitor 123. Because the given potential difference $(V = 12.5V)$ is applied across the actual combination of three capacitors in Fig. *a*, it is also applied across C_{123} in Figs. *d* and *e*. Thus, $(q = CV)$ gives us:

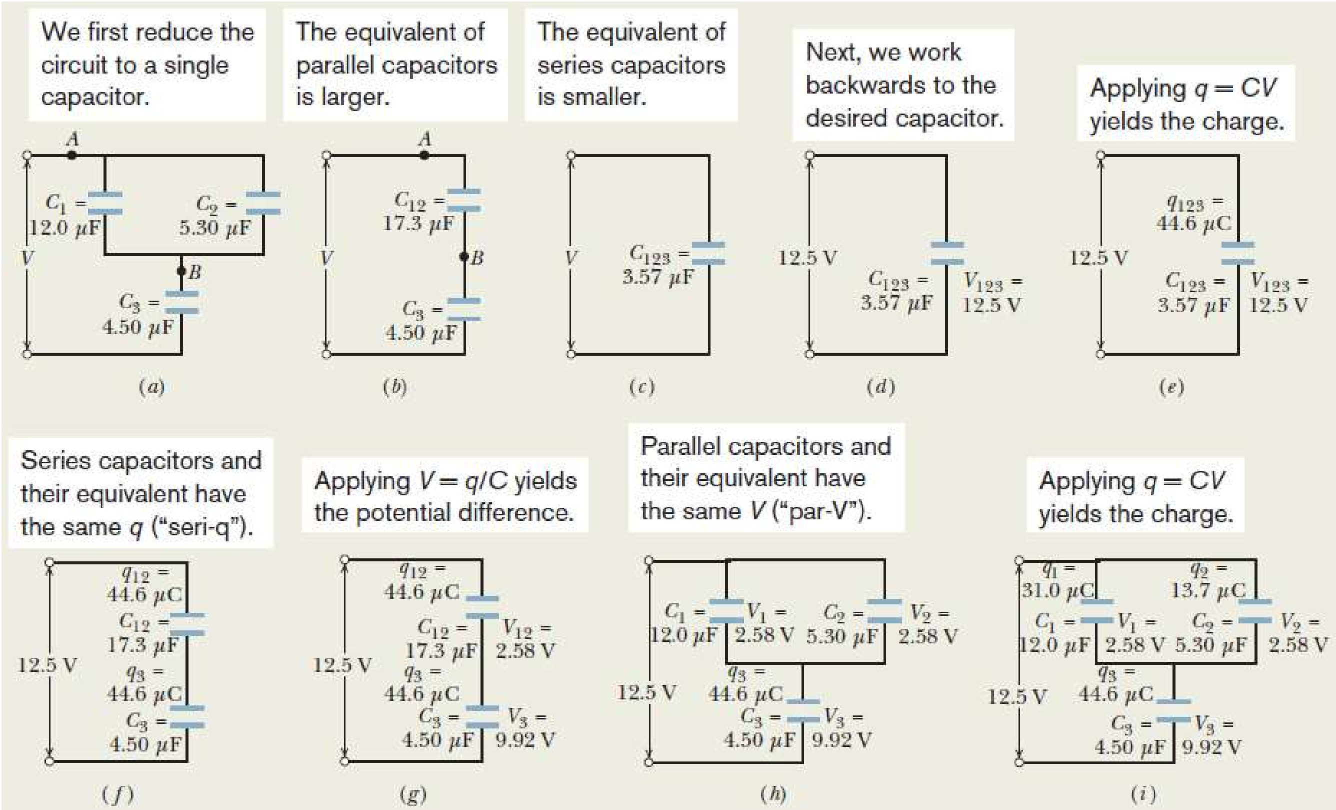
$$q_{123} = C_{123}V = (3.57\mu F)(12.5V) = 44.6\mu C$$

The series capacitors 12 and 3 in Fig. *b* each have the same charge as their equivalent capacitor 123 (Fig. *f*). Thus, capacitor 12 has charge $q_{12} = q_{123} = 44.6\mu C$. From Fig. *g*, the potential difference across capacitor 12 must be

$$V_{12} = \frac{q_{12}}{C_{12}} = \frac{44.6\mu C}{17.3\mu F} = 2.58V$$

The parallel capacitors 1 and 2 each have the same potential difference as their equivalent capacitor 12 (Fig. *h*). Thus, capacitor 1 has potential difference $V_1 = V_{12} = 2.58V$. and, from Fig. *i*, the charge on capacitor 1 must be

$$q_1 = C_1V_1 = (12.0\mu F)(2.58V) = 31.0\mu C$$


We first reduce the circuit to a single capacitor.
The equivalent of parallel capacitors is larger.
The equivalent of series capacitors is smaller.
Next, we work backwards to the desired capacitor.
Applying q = CV yields the charge.
Series capacitors and their equivalent have the same q ("seri-q").
Applying V = q/C yields the potential difference.
Parallel capacitors and their equivalent have the same V ("par-V").
Applying q = CV yields the charge.
C1 = 12.0 μF
C2 = 5.30 μF
C3 = 4.50 μF
C12 = 17.3 μF
C123 = 3.57 μF
V123 = 12.5 V
q123 = 44.6 μC
q12 = 44.6 μC
q3 = 44.6 μC
V12 = 2.58 V
V3 = 9.92 V
V1 = 2.58 V
V2 = 2.58 V
q1 = 31.0 μC
q2 = 13.7 μC
12.5 V
(a) (b) (c) (d) (e) (f) (g) (h) (i)

Electric-Field Energy

Sunday, 11 April, 2021 18:17

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- ☑ H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

- What is the magnitude of the electric field required to store 1.00 J of electric potential energy in a volume of 1.00 m^3 in vacuum?
- If the field magnitude is 10 times larger than that, how much energy is stored per cubic meter?

- We use the relationship between the electric field magnitude E and the energy density u.

$$u = \frac{U}{V} = \frac{1}{2}\epsilon_\circ E^2$$

$$E = \sqrt{\frac{2U}{\epsilon_\circ V}}$$

$$= \sqrt{\frac{2 \times 1}{8 \cdot 85 \times 10^{-12} \times 1}} = 4 \cdot 75 \times 10^5 \, V/m$$

- u is proportional to E^2 . If E increases by a factor of 10, u increases by a factor of $10^2 = 100$, so the energy density becomes $u = 100 \, J/m^3$.

Potential energy and energy density of an electric field

Tuesday, 2 February, 2021 18:25

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- [ ] R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- [x] J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- [ ] H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- [ ] H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

An isolated conducting sphere whose radius R is $6.85\ cm$ has a charge $q = 1.25\ nC$.

- How much potential energy is stored in the electric field of this charged conductor?
- What is the energy density at the surface of the sphere?

- Substituting $(C = 4\pi\epsilon_\circ R)$ into $(U = q^2/2C)$ gives us

$$U = \frac{q^2}{2C} = \frac{q^2}{8\pi\epsilon_\circ R} = \frac{(1.25 \times 10^{-9})^2}{(8\pi)(8.85 \times 10^{-12})(0.0685)}$$

$$= 1.03 \times 10^{-7} J = 103 nJ$$

- The density u of the energy stored in an electric field depends on the magnitude E of the field, according to $(u = \frac{1}{2}\epsilon_\circ E^2)$.

Here we must first find E at the surface of the sphere, as given by:

$$E = \frac{1}{4\pi\epsilon_\circ}\frac{q}{R^2}$$

The energy density is then

$$u = \frac{1}{2}\epsilon_\circ E^2 = \frac{q^2}{32\pi^2\epsilon_\circ R^4}$$

$$= \frac{(1.25 \times 10^{-9})^2}{(32\pi^2)(8.85 \times 10^{-12})(0.0685)^4}$$

$$= 2.54 \times 10^{-5} J/m^3 = 25.4 \mu J/m^3.$$

Rewiring Two Charged Capacitors

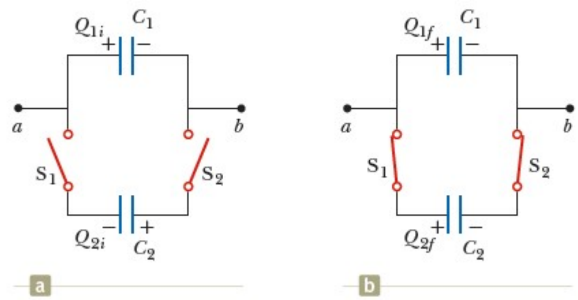
Tuesday, 2 February, 2021 16:42

Lecturer: Mustafa Al-Zyout, Philadelphia University, Jordan.

- R. A. Serway and J. W. Jewett, Jr., *Physics for Scientists and Engineers*, 9th Ed., CENGAGE Learning, 2014.
- J. Walker, D. Halliday and R. Resnick, *Fundamentals of Physics*, 10th ed., WILEY,2014.
- H. D. Young and R. A. Freedman, *University Physics with Modern Physics*, 14th ed., PEARSON, 2016.
- H. A. Radi and J. O. Rasmussen, *Principles of Physics For Scientists and Engineers*, 1st ed., SPRINGER, 2013.

Two capacitors C_1 and C_2 (where $C_1 > C_2$) are charged to the same initial potential difference ΔV_i . The charged capacitors are removed from the battery, and their plates are connected with opposite polarity. The switches S_1 and S_2 are then closed.

- Find the final potential difference ΔV_f between a and b after the switches are closed.
- Find the total energy stored in the capacitors before and after the switches are closed and determine the ratio of the final energy to the initial energy.



SOLUTION

(A) When the switches are closed, the charge on the system will redistribute between the capacitors until both capacitors have the same potential difference. Because $C_1 > C_2$, more charge exists on C_1 than on C_2, so the final configuration will have positive charge on the left plates as shown in Figure b.

In Figure b, it might appear as if the capacitors are connected in parallel, but there is no battery in this circuit to apply a voltage across the combination. Therefore, we *cannot* categorize this problem as one in which capacitors are connected in parallel. We *can* categorize it as a problem involving an isolated system for electric charge. The left-hand plates of the capacitors form an isolated system because they are not connected to the right-hand plates by conductors.

Write an expression for the total charge on the left-hand plates of the system before the switches are closed, noting that a negative sign for Q_{2i} is necessary because the charge on the left plate of capacitor C_2 is negative:

$$(1)Q_i = Q_{1i} + Q_{2i} = C_1\Delta V_i - C_2\Delta V_i = (C_1 - C_2)\Delta V_i$$

After the switches are closed, the charges on the individual capacitors change to new values Q_{1f} and Q_{2f} such that the potential difference is again the same across both capacitors, with a value of ΔV_f. Write an expression for the total charge on the left-hand plates of the system after the switches are closed:

$$(2)Q_f = Q_{1f} + Q_{2f} = C_1\Delta V_f + C_2\Delta V_f = (C_1 + C_2)\Delta V_f$$

Because the system is isolated, the initial and final charges on the system must be the same. Use this condition and Equations (1) and (2) to solve for ΔV_f:

$$Q_f = Q_i \rightarrow (C_1 + C_2)\Delta V_f = (C_1 - C_2)\Delta V_i$$

$$(3)\Delta V_f = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i$$

(B) find an expression for the total energy stored in the capacitors before the switches are closed:

$$(4) U_i = \frac{1}{2}C_1(\Delta V_i)^2 + \frac{1}{2}C_2(\Delta V_i)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V_i)^2$$

Write an expression for the total energy stored in the capacitors after the switches are closed:

$$U_f = \frac{1}{2}C_1(\Delta V_f)^2 + \frac{1}{2}C_2(\Delta V_f)^2 = \frac{1}{2}(C_1 + C_2)(\Delta V_f)^2$$

Use the results of part (A) to rewrite this expression in terms of ΔV_i:

$$(5) U_f = \frac{1}{2}(C_1 + C_2)\left[\left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i\right]^2 = \frac{1}{2}\frac{(C_1 - C_2)^2(\Delta V_i)^2}{C_1 + C_2}$$

Divide Equation (5) by Equation (4) to obtain the ratio of the energies stored in the system:

$$\frac{U_f}{U_i} = \frac{\frac{1}{2}(C_1 - C_2)^2(\Delta V_i)^2/(C_1 + C_2)}{\frac{1}{2}(C_1 + C_2)(\Delta V_i)^2}$$

$$(6) \frac{U_f}{U_i} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$

The ratio of energies is *less* than unity, indicating that the final energy is *less* than the initial energy. At first, you might think the law of energy conservation has been violated, but that is not the case. The “missing” energy is transferred out of the system by the mechanism of electromagnetic waves. Therefore, this system is isolated for electric charge, but nonisolated for energy.

What if the two capacitors have the same capacitance? What would you expect to happen when the switches are closed?

Answer: Because both capacitors have the same initial potential difference applied to them, the charges on the capacitors have the same magnitude. When the capacitors with opposite polarities are connected together, the equal-magnitude charges should cancel each other, leaving the capacitors uncharged.

Let’s test our results to see if that is the case mathematically. In Equation (1), because the capacitances are equal, the initial charge Q_i on the system of left-hand plates is zero. Equation (3) shows that $\Delta V_f = 0$, which is consistent with uncharged capacitors. Finally, Equation (5) shows that $U_f = 0$, which is also consistent with uncharged capacitors.